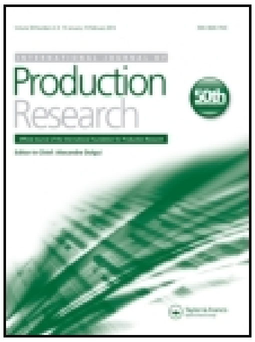

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A piecewise nonlinear model for a production system under maintenance, trade credit and limited warehouse space

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Small and medium size manufacturers exist commonly in developing countries. Limited warehouse space is a characteristic in small and medium size manufacturers. In today's business environment, many upstream companies offer a credit period to downstream companies. In production system, a manufacturer usually adopts the system maintenance when the system is in the out-of-control state. Therefore, this paper considers a production-inventory model for a manufacturer under system maintenance, trade credit and limited warehouse space. The objective is to determine the optimal production run time to minimise the total cost. We develop an algorithm based on several theorems for solving the problem described. We provide several examples to illustrate the solution procedure and discuss how system parameters affect the manufacturer's decision behaviour. Computational analysis demonstrates that the results of the proposed model are consistent with economic insights.

Keywords: piecewise nonlinear; inventory; production; maintenance; limited warehouse space; trade credit

1. Introduction

The inventory model is the most widely used model in inventory management. For example, IBM has developed a parts inventory management system, which is based on the EOQ model, to provide prompt and reliable customer service. Recently, Sana (2012a), (2013) and Pal and Sana (2012a) considered the inventory model under different situations.

The traditional inventory model tacitly assumes that the payment must be made to the vendor for the items immediately after the buyer receives the products. In practice, the vendor often provides forward financing to the buyer. This means that the vendor allows the buyer a certain fixed period (credit period) in which to settle the amount owed, and does not charge any interest on the amount owed during this period. For example, Wal-Mart, the largest retailer in the world, uses trade credit as a larger source of capital than bank loans; trade credit for Wal-Mart is eight times the amount of capital invested by shareholders. Over the years, a number of studies have been published that deal with the economic order quantity problems under conditions of permissible delay in payments, i.e., credit period in our research.

Goyal (1985) is the first paper which examined the effect of the credit period on the optimal inventory policy. So far, a lot of papers have appeared in the literature dealing with a variety of trade credit situations. Teng (2002) amended the model of Goyal (1985) by considering the difference between unit price and unit cost. Huang (2003) considered not only that the supplier offers a credit period to the retailer but also that the retailer offers a credit period to his customer. Huang and Chung (2003) and Ouyang, Chang, and Teng (2005) considered the ordering policy under cash discount and payment delay. Chung and Huang (2006) considered an EOQ model to allow items with imperfect quality under permissible delay in payments. Sheen and Tsao (2007) discussed channel coordination under trade credit with freight cost having quantity discounts. Ouyang, Ho, and Su (2008) demonstrated that significant profit increase for the entire supply chain can be achieved by linking both trade credit and freight rate policies. Tsao and Sheen (2008) determined the dynamic pricing, promotion and replenishment policies for a deteriorating item under trade credits. Sana and Chaudhuri (2008) developed a deterministic EOQ model with delays in payments and price–discount offers. Ouyang et al. (2009) considered deteriorating items with partially permissible delay in payments linked to order quantity. Tsao (2010) determined two-phase pricing and inventory decisions for deteriorating and fashion goods under trade credit conditions. Balkhi (2011) considered an economic ordering policy with deteriorating items under different supplier trade credits for finite horizon case. Tsao and Sheen (2012) developed a multi-item supply chain with credit period and freight cost discounts. Therefore, the issue of trade credit is very popular to these field researchers.

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Most of the trade credit papers considered EOQ model. To date, Chung and Huang (2003), Ouyang, Chang, and Teng (2006), Huang (2007), Liao (2008), Teng and Chang (2009), Tsao (2009), Chang, Teng, and Chern (2010) and Tsao, Chen, and Zhang (2013) considered economic production quantity (EPQ) model under trade credit. Pal, Sana, and Chaudhuri (2013a) investigated the optimal replenishment lot size of supplier and optimal production rate of manufacturer under a three-level supply chain. Based on their great works, this paper considers the imperfect production system. This means the production system may produce defective products. For imperfect production system, Porteus (1986) was one of the first to consider the situation where the production process may shift from an 'in-control' state to an 'out-of-control' state with a given probability each time it produces an item. In a similar type of work, Rosenblatt and Lee (1986) found, by assuming exponential shift distribution, that the optimal manufacturing quantity for an imperfect production system is smaller than that for a system with no defects. But, they did not consider trade credit and they assumed there is no defective product to produce in the 'in-control' state.

Recently, Wee and Widyadana (2012) developed EPQ models for deteriorating items with rework and stochastic preventive maintenance time Sana (2012b) deals with an imperfect production system with allowable shortages due to regular preventive maintenance for products sold with free minimal repair warranty. Wee, Wang, and Yang (2013) developed an EPQ model and solution procedure for imperfect items with shortage and screening constraint. Pan et al. (2012) developed an integrated EPQ model by combining the concepts of statistical process control and maintenance. In their model, a control chart is adopted to monitor the whole process with Taguchi's loss function estimating the quality cost of each condition. Other similar studies, Sana (2010a, 2010b), Sana and Chaudhuri (2010), Sarkar, Sana, and Chaudhuri (2011) and Pal, Sana, and Chaudhuri (2013b), considered the imperfect production system Pal, Sana, and Chaudhuri (2012b, 2012c) considered the reworkable items and Sana (2011), Roy, Sana, and Chaudhuri (2011), Pal, Sana, and Chaudhuri (2013c) and Sana (2012c) considered imperfect quality items. In our paper, the system maintenance activity is considered in the inventory model with trade credit and limited warehouse space to cope with more realistic situations.

Also, trade credit policy induces a company to store more items. In this case, the company may rent a warehouse to store the exceeding items when the quantity is larger than companies' warehouse capacity. This is common in small and medium size company. Especially in developing countries, they rely on small and medium size company to compete in international market. In Taiwan, about 98% are small and medium size companies. Huang (2006) and Teng and Chang (2009) considered EOQ model under trade credit and limited storage space. To the best of our knowledge, our paper is the first to incorporate the limited warehouse space, trade credit and system maintenance into EPQ model simultaneously. The objective is to determine the optimal production run time to minimise the total cost. This paper provides an algorithm based on several theorems for solving the problem. From computational analysis, we discuss how the system parameters affect the manufacturer's decision and cost. The purpose of this paper is to relax several assumptions, such as system maintenance, trade credit and limited warehouse space, to cope with more practical situations.

2. Notations and assumptions

The following notations are used in this paper:

r	retail price per unit
c	purchase cost per unit
D	demand rate or delivery rate
P	production rate
T_D	cycle time
T_P	production run time, $T_P = DT_D/P$
K	set-up cost per set-up
h	inventory holding cost per unit in manufacturer's warehouse
H	inventory holding cost per unit in rented warehouse, $H > h$
s	defective cost, the cost incurred by producing a defective item
m	machine maintenance cost
α_I	percentage of defective products in the in-control state
α_O	percentage of defective products in the out-of-control state, $\alpha_O > \alpha_I$
τ	time when the production process shifts
Ie	interest earned per dollar per year
Ip	interest paid per dollar per year

M credit period
W manufacturer's warehouse capacity
T_O the length of the rented warehouse time

$$T_O = \begin{cases} \dfrac{T_P(P-D)-W}{(P-D)D/P} = \dfrac{T_P P(P-D)-WP}{(P-D)D}, & \text{if } T_P(P-D) > W \\ 0, & \text{if } T_P(P-D) \leqslant W \end{cases}$$

The mathematical model in this paper is developed on the basis of the following assumptions:

(1) The production rate P is larger than demand rate D.
(2) The production process may shift from an 'in-control' state to an 'out-of-control' state during a production run.
(3) The elapsed time until the production process shifts, τ, is a random variable and assumed to be exponentially distributed with a mean of $1/\lambda$.
(4) System maintenance will be taken into account if the system is in the 'out-of-control' state. After the maintenance, the system can be restored to the 'in-control' state. And machine maintenance can be completed before the next production run.
(5) If the production quantity is larger than manufacturer's warehouse capacity W, the manufacturer will rent the warehouse to store these exceeding items.
(6) The unit retail price of the products sold during the credit period is deposited in an interest bearing account with the rate Ie. At the end of this period, the credit is settled and the retailer starts paying the interest paid for the items in stock with the rate Ip.
(7) The retail price is larger than the purchase cost, ie $r \geq c$.
(8) A non-defective item can be produced immediately to replace a defective item which was produced from the system.

3. Model formulation

If the production run time T_P is larger than $\frac{W}{P-D}$, the manufacturer need to rent the warehouse to store these exceeding items; if $T_P \leq \frac{W}{P-D}$, the manufacturer can store all the items in his own warehouse. The total annual cost consists of the following elements. Since $\frac{W}{P-D}$ is a judgment standard, three cases may arise: (1) $M > \frac{D}{P}M \geq \frac{W}{P-D}$, (2) $M \geq \frac{W}{P-D} > \frac{D}{P}M$, (3) $\frac{W}{P-D} > M$.

Case 1 When $M > \frac{D}{P}M \geq \frac{W}{P-D}$

(1) Annual set-up cost = $K/T_D = KD/(PT_P)$.
(2) There are two cases in annual inventory holding cost.

(i) when $T_P > \frac{W}{P-D}$.
In this case, production quantity is larger than manufacturer's warehouse capacity W. So, the manufacturer needs to rent the warehouse to store the exceeding items. Hence, annual inventory holding cost = annual inventory holding of rented warehouse + annual inventory holding cost of the warehouse capacity W.

$$\text{Annual inventory holding cost} = \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P}, \quad (1)$$

(ii) when $T_P \leqslant \frac{W}{P-D}$.
In this case, the order quantity is not larger than manufacturer's warehouse capacity. So, it is not necessary for the manufacturer to rent warehouse to store items. Hence,

$$\text{Annual inventory holding cost} = \frac{h(P-D)T_P}{2}. \quad (2)$$

(3) There are three cases in interest earned per year.
The unit retail price of the products sold during the credit period is to earn interest with the rate Ie.
(i) when $T_P \geq M$, as shown in Figure 1(a).

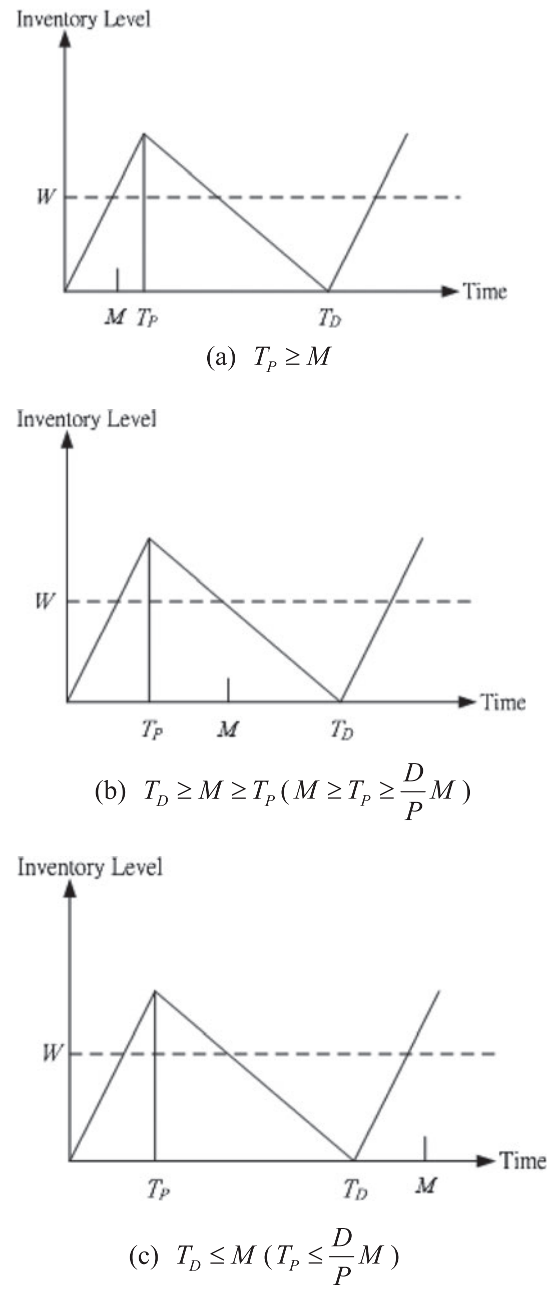

Figure 1. Inventory level for each case.

$$\text{Annual interest earned} = \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P}, \tag{3}$$

(ii) when $T_D \geq M \geq T_P$ $(M \geqslant T_P \geqslant \frac{D}{P}M)$, as shown in Figure 1(b).

$$\text{Annual interest earned} = \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P}, \tag{4}$$

(iii) when $T_D \leq M$ $(T_P \leqslant \frac{D}{P}M)$, as shown in Figure 1(c).

$$\text{Annual interest earned} = \frac{r \cdot Ie \cdot P \cdot T_P}{2} + r \cdot Ie \cdot D \cdot \left(M - \frac{P \cdot T_P}{D}\right). \tag{5}$$

(4) There are three cases in interest paid per year.
When the credit period M is shorter than the cycle time T_D, the products still in inventory have to be financed with the rate Ip after the payment is settled.

(i) When $T_P \geq M$, as shown in Figure 1(a).

$$\text{Annual interest paid} = c \cdot Ip \cdot \left[\frac{PT_P}{2}\left(1 - \frac{D}{P}\right) - \frac{(P-D)D \cdot M^2}{2PT_P}\right]. \tag{6}$$

(ii) When $T_D \geq M \geq T_P$ ($M \geqslant T_P \geqslant \frac{D}{P}M$), as shown in Figure 1(b).

$$\text{Annual interest paid} = \frac{c \cdot Ip \cdot D^2 \cdot (PT_P/D - M)^2}{2PT_P}, \tag{7}$$

(iii) When $T_D \leq M(T_P \leqslant \frac{D}{P}M)$, as shown in Figure 1(c).

Annual interest paid = 0. In this case, no interest charge is paid for the items.

(5) Annual defective cost is the product of defective cost per unit and the number of defective items:

The number of defective items N in each production run is

$$N = \begin{cases} \alpha_I PT_P & \text{if } \tau \geqslant T_P, \\ \alpha_I P\tau + \alpha_O P(T_P - \tau) & \text{if } \tau < T_P. \end{cases} \tag{8}$$

Then, the expected number of defective items in each production run $E(N)$ is

$$E(N) = \int_{T_P}^{\infty} \alpha_I PT_P \lambda e^{-\lambda\tau} \mathrm{d}\tau + \int_0^{T_P} [\alpha_I P\tau + \alpha_O P(T_P - \tau)]\lambda e^{-\lambda\tau} \mathrm{d}\tau = -P\alpha_I \frac{e^{-\lambda T_P} - 1}{\lambda} + P\alpha_O\left(T_P + \frac{e^{-\lambda T_P} - 1}{\lambda}\right). \tag{9}$$

And the annual defective cost is $\dfrac{sDE(N)}{PT_P} = sD\left[-\alpha_I \dfrac{e^{-\lambda T_P} - 1}{\lambda T_P} + \alpha_O\left(1 + \dfrac{e^{-\lambda T_P} - 1}{\lambda T_P}\right)\right]$.

If λ is very small, it is reasonable to use McClaurin series to approximate $e^{-\lambda T_P} \approx 1 - \lambda T_P + (\lambda T_P)^2/2$. The idea of approximation to simplify the mathematical calculation has been used in many studies Therefore, we can rewrite the annual defective cost:

$$\frac{sDE(N)}{PT_P} \approx sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right]. \tag{10}$$

(6) Annual machine maintenance cost is the product of machine maintenance cost per time and times of annual machine maintenance:

The times annual machine maintenance is $D(1 - e^{-\lambda T_P})/PT_P \approx D(\lambda - \lambda^2 T_P/2)/P$.

$$\text{Annual machine maintenance cost} = mD(\lambda - \lambda^2 T_P/2)/P. \tag{11}$$

Therefore, total annual cost $TC(T_P)$ is

$$TC(T_P) = \begin{cases} TC_1(T_P), & \text{if } T_P \geqslant M, \\ TC_2(T_P), & \text{if } M \geqslant T_P \geqslant \dfrac{D}{P}M, \\ TC_3(T_P), & \text{if } \dfrac{D}{P}M \geqslant T_P \geqslant \dfrac{W}{P-D}, \\ TC_6(T_P), & \text{if } T_P \leqslant \dfrac{W}{P-D}, \end{cases} \tag{12}$$

where

$$TC_1(T_P) = \frac{KD}{PT_P} + \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right)$$
$$+ sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + c \cdot Ip \cdot \left[\frac{PT_P}{2}\left(1 - \frac{D}{P}\right) - \frac{(P-D)D \cdot M^2}{2PT_P}\right], \quad (13)$$

$$TC_2(T_P) = \frac{KD}{PT_P} + \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right)$$
$$+ sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + \frac{c \cdot Ip \cdot D^2 \cdot (PT_P/D - M)^2}{2PT_P}, \quad (14)$$

$$TC_3(T_P) = \frac{KD}{PT_P} + \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right)$$
$$+ sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot P \cdot T_P}{2} - r \cdot Ie \cdot D \cdot \left(M - \frac{P \cdot T_P}{D}\right) \quad (15)$$

$$TC_6(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot P \cdot T_P}{2} - r \cdot Ie \cdot D \cdot \left(M - \frac{P \cdot T_P}{D}\right) \quad (16)$$

Since $TC_1(t) = TC_2(t)$, $TC_2(\frac{D}{P}M) = TC_3(\frac{D}{P}M)$, and $TC_3(\frac{W}{P-D}) = TC_6(\frac{W}{P-D})$, $TC(T_P)$ is continuous and well defined on $T_p > 0$. All $TC_1(T_P)$, $TC_2(T_P)$, $TC_3(T_P)$, $TC_6(T_P)$ and $TC(T_P)$ are defined on $T_p > 0$.

Case 2 When $M \geqslant \frac{W}{P-D} > \frac{D}{P}M$.

Total annual cost $TC(T_P)$ is

$$TC(T_P) = \begin{cases} TC_1(T_P), & \text{if } T_P \geqslant M, \\ TC_2(T_P), & \text{if } M \geqslant T_P \geqslant \dfrac{W}{P-D}, \\ TC_5(T_P), & \text{if } \dfrac{W}{P-D} \geqslant T_P \geqslant \dfrac{D}{P}M, \\ TC_6(T_P), & \text{if } T_P \leqslant \dfrac{D}{P}M, \end{cases} \quad (17)$$

where

$$TC_1(T_P) = \frac{KD}{PT_P} + \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right)$$
$$+ sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + c \cdot Ip \cdot \left[\frac{PT_P}{2}\left(1 - \frac{D}{P}\right) - \frac{(P-D)D \cdot M^2}{2PT_P}\right], \quad (18)$$

$$TC_2(T_P) = \frac{KD}{PT_P} + \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right)$$
$$+ sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + \frac{c \cdot Ip \cdot D^2 \cdot (PT_P/D - M)^2}{2PT_P}, \quad (19)$$

$$TC_5(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + \frac{c \cdot Ip \cdot D^2 \cdot (PT_P/D - M)^2}{2PT_P}, \quad (20)$$

$$TC_6(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot P \cdot T_P}{2} - r \cdot Ie \cdot D \cdot \left(M - \frac{P \cdot T_P}{D}\right). \tag{21}$$

Since $TC_1(M) = TC_2(M)$, $TC_2\left(\frac{W}{P-D}\right) = TC_5\left(\frac{W}{P-D}\right)$, and $TC_5\left(\frac{D}{P}M\right) = TC_6\left(\frac{D}{P}M\right)$, $TC(T_P)$ is continuous and well defined on $T_p > 0$. All $TC_1(T_P)$, $TC_2(T_P)$, $TC_5(T_P)$, $TC_6(T_P)$ and $TC(T_P)$ are defined on $T_p > 0$.

Case 3 When $\frac{W}{P-D} > M$.
Total annual cost $TC(T_P)$ is

$$TC(T_P) = \begin{cases} TC_1(T_P), & \text{if } T_P \geqslant \dfrac{W}{P-D}, \\ TC_4(T_P), & \text{if } \dfrac{W}{P-D} \geqslant T_P \geqslant M, \\ TC_5(T_P), & \text{if } M \geqslant T_P \geqslant \dfrac{D}{P}M, \\ TC_6(T_P), & \text{if } T_P \leqslant \dfrac{D}{P}M, \end{cases} \tag{22}$$

where

$$\begin{aligned} TC_1(T_P) = {} & \frac{KD}{PT_P} + \frac{H[(P-D)T_P - W]^2}{2(P-D)T_P} + \frac{hW^2 D}{2(P-D)PT_P} + \frac{h[(P-D)T_P - W]W}{(P-D)T_P} + \frac{hW^2}{2PT_P}\frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) \\ & + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + c \cdot Ip \cdot \left[\frac{PT_P}{2}\left(1 - \frac{D}{P}\right) - \frac{(P-D)D \cdot M^2}{2PT_P}\right], \end{aligned} \tag{23}$$

$$\begin{aligned} TC_4(T_P) = {} & \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} \\ & + c \cdot Ip \cdot \left[\frac{PT_P}{2}\left(1 - \frac{D}{P}\right) - \frac{(P-D)D \cdot M^2}{2PT_P}\right], \end{aligned} \tag{24}$$

$$TC_5(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + \frac{c \cdot Ip \cdot D^2 \cdot (PT_P/D - M)^2}{2PT_P}, \tag{25}$$

$$TC_6(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot P \cdot T_P}{2} - r \cdot Ie \cdot D \cdot \left(M - \frac{P \cdot T_P}{D}\right). \tag{26}$$

Since $TC_1(\frac{W}{P-D}) = TC_4(\frac{W}{P-D})$, $TC_4(M) = TC_5(M)$, and $TC_5(\frac{D}{P}M) = TC_6(\frac{D}{P}M)$, TC($T_P$) is continuous and well defined on $T_p > 0$. All $TC_1(T_P)$, $TC_4(T_P)$, $TC_5(T_P)$, $TC_6(T_P)$ and $TC(T_P)$ are defined on $T_p > 0$.

4. Solution procedure

The objective of this section is to develop a solution procedure to determine the optimal production run time while still minimising the total cost. We discuss solution lemmas and present an algorithm that can be utilised to solve the problem.

Case 1 When $M > \frac{D}{P}M \geqslant \frac{W}{P-D}$
The second-order derivative of $TC_1(T_P)$ with respect to T_P is

$$\frac{d^2 TC_1(T_P)}{dT_P^2} = \frac{D(P-D)\left\{2K - M^2[cIp(P-D) + DIer]\right\} + (H-h)PW^2}{(P-D)PT_P^3}. \tag{27}$$

If $2K > M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, then $TC_1(T_P)$ is a convex function of T_P. Therefore, there exists a production run time $T_{P,1}$ which minimises $TC_1(T_P)$ as follows. Solving $\frac{dTC_1(T_P)}{dT_P} = 0$, we obtain

$$T_{P,1} = \sqrt{\frac{(P-D)D\{2K - M^2[cIp(P-D) + DIer]\} + (H-h)PW^2}{(P-D)\left[(H+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right]}}. \tag{28}$$

Equation (28) gives the optimal value of $T_{P,1}$ for the case when $T_P \geq M$. We substitute Equation (28) into $T_P \geq M$ to obtain that $T_{P,1} \geqslant M$ if and only if

$$2K \geqslant \frac{M^2}{D}\left[(H+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right] + M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{(P-D)D}. \tag{29}$$

Let $G_1 = \frac{M^2}{D}\left[(H+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right] + M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{(P-D)D}$. From $G_1 - \{M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}\} = \frac{M^2}{D}\left[(H+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right]$ and λ is very small, we know that $G_1 > M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$. Then, we have the following lemma base on above analysis:

Lemma 1:

(a) If $2K \geq G_1$, there exists an optimal production run time $T_{P,1}$ (in Equation (28)).
(b) If $G_1 > 2K > M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, there is no feasible solution in this case.
(c) If $2K \leqslant M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, the minimal value of $TC_1(T_P)$ occurs at $T_P = M$.

Proof:

(a) If 2 $K \geq G_1$, $TC_1(T_P)$ is a convex function of T_P and $T_{P,1}$ in Equation (28) satisfies $T_P \geq M$. Therefore, $T_{P,1}$ in Equation (28) is the minimum point in $TC_1(T_P)$.
(b) If $G_1 > 2K > M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, there is no solution in the range of $T_P \geq t$. So, there is no feasible solution in this case.
(c) If $2K \leqslant M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, $TC_1(T_P)$ is a concave function of T_P and $\frac{dTC_1(T_P)}{dT_P} > 0$. This means $TC_1(T_P)$ is a increasing function of T_P where $T_P \in [M, \infty)$. Therefore, the minimal value of $TC_1(T_P)$ occurs at T_P=M. -

□

Similarly, the second-order derivative of $TC_2(T_P)$ with respect to T_P is

$$\frac{d^2TC_2(T_P)}{dT_P^2} = \frac{D(P-D)[2K + DM^2(cIp - rIe)] + (H-h)PW^2}{(P-D)PT_P^3} > 0. \tag{30}$$

This means $TC_2(T_P)$ is a convex function of T_P. Then, there exists a production run time $T_{P,2}$ which minimises $TC_2(T_P)$ as follows. Solving $\frac{dTC_2(T_P)}{dT_P} = 0$, we obtain

$$T_{P,2} = \sqrt{\frac{D(P-D)[2K + DM^2(cIp - rIe)] + (H-h)PW^2}{(P-D)\left\{(H+cIp)P^2 - D\left[HP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\}}}. \tag{31}$$

Equation (31) gives the optimal value of $T_{P,2}$ for the case when $M \geqslant T_P \geqslant \frac{D}{P}M$. We substitute Equation (31) into $M \geqslant T_P \geqslant \frac{D}{P}M$ to obtain that $M \geqslant T_{P,2} \geqslant \frac{D}{P}M$ if and only if

$$\begin{aligned} &\frac{M^2}{D}\left[(H+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right] + M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{(P-D)D} \\ &\geqslant 2K \geqslant \frac{DM^2}{P^2}\left\{(H+rIe)P^2 - D[HP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2]\right\} - \frac{(H-h)PW^2}{(P-D)D}. \end{aligned} \tag{32}$$

Let $G_2 = \frac{DM^2}{P^2}\left\{(H+rIe)P^2 - D\left[HP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\} - \frac{(H-h)PW^2}{(P-D)D}$. Then we have the following lemma base on above analysis:

Lemma 2:

(a) If $2K > G_1$, there is no feasible solution in this case.
(b) If $G_1 \geq 2K \geq G_2$, there exists an optimal production run time $T_{P,2}$ (in Equation (31)).
(c) If $2K < G_2$, there is no feasible solution in this case.

The second-order derivative of $TC_3(T_P)$ with respect to T_P is

$$\frac{d^2TC_3(T_P)}{dT_P^2} = \frac{2DK(P-D)+(H-h)PW^2}{(P-D)PT_P^3} > 0. \tag{33}$$

This means $TC_3(T_P)$ is a convex function of T_P. There exists a production run time $T_{P,3}$ which minimises $TC_3(T_P)$ as follows. Solving $\frac{dTC_3(T_P)}{dT_P} = 0$, we obtain

$$T_{P,3} = \sqrt{\frac{2DK(P-D)+(H-h)PW^2}{(P-D)\{(H+rIe)P^2 - D[HP - Ps(\alpha_O-\alpha_I)\lambda + m\lambda^2]\}}}. \tag{34}$$

Equation (34) gives the optimal value of $T_{P,3}$ for the case when $\frac{D}{P}M \geqslant T_P \geqslant \frac{W}{P-D}$. We substitute Equation (34) into $\frac{D}{P}M \geqslant T_P \geqslant \frac{W}{P-D}$ to obtain that $\frac{D}{P}M \geqslant T_{P,3} \geqslant \frac{W}{P-D}$ if and only if

$$\frac{DM^2}{P^2}\{(H+rIe)P^2 - D[HP - Ps(\alpha_O-\alpha_I)\lambda + m\lambda^2]\} - \frac{(H-h)PW^2}{(P-D)D}$$
$$\geqslant 2K \geqslant \frac{W^2}{D(P-D)^2}\{(h+rIe)P^2 - D[hP - Ps(\alpha_O-\alpha_I)\lambda + m\lambda^2]\}. \tag{35}$$

Let $G_3 = \frac{W^2}{D(P-D)^2}\{(h+rIe)P^2 - D[hP - Ps(\alpha_O-\alpha_I)\lambda + m\lambda^2]\}$. Then we have the following lemma base on above analysis:

Lemma 3:

(a) If $2K > G_2$, there is no feasible solution in this case.
(b) If $G_2 \geq 2K \geq G_3$, there exists an optimal production run time $T_{P,3}$ (in Equation (34)).
(c) If $2K < G_3$, there is no feasible solution in this case.

At last, the second-order derivative of $TC_6(T_P)$ with respect to T_P is

$$\frac{d^2TC_6(T_P)}{dT_P^2} = \frac{2DK}{PT_P^3} > 0. \tag{36}$$

This means $TC_6(T_P)$ is a convex function of T_P. There exists a production run time $T_{P,6}$ which minimises $TC_3(T_P)$ as follows. Solving $\frac{dTC_6(T_P)}{dT_P} = 0$, we obtain

$$T_{P,6} = \sqrt{\frac{2DK}{(H+rIe)P^2 - D[HP - Ps(\alpha_O-\alpha_I)\lambda + m\lambda^2]}}. \tag{37}$$

Equation (37) gives the optimal value of $T_{P,6}$ for the case when $T_P \leqslant \frac{W}{P-D}$. We substitute Equation (37) into $T_P \leqslant \frac{W}{P-D}$ to obtain that $T_{P,6} \leqslant \frac{W}{P-D}$ if and only if

$$2K \leqslant \frac{W^2}{D(P-D)^2}\{(h+rI_e)P^2 - D[hP - Ps(\alpha_O-\alpha_I)\lambda + m\lambda^2]\}. \tag{38}$$

Then, we have the following lemma base on above analysis:

Lemma 4:

(a) If $2K > G_3$, there is no feasible solution in this case.
(b) If $2K \leq G_3$, there exists an optimal production run time $T_{P,6}$ (in Equation (37)).

From Equations (32) and (35), we know that $G_1 \geq G_2 \geq G_3$. Let $G_\alpha = M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, we have Theorem 1 based on above analysis:

Theorem 1:

(1) If $2K \geq G_1$, then $TC(T_P^*) = TC_1(T_{P,1})$.
(2) If $G_\alpha > G_2$, then

(a) If $G_1 \geq 2K \geq G_\alpha \geq G_2$, then $TC(T_P^*) = TC_2(T_{P,2})$.
(b) If $G_2 \geq 2K \geq G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_3(T_{P,3})\}$.
(c) If $2K \leq G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

(3) If $G_\alpha = G_2$, then
(a) If $G_1 \geq 2K \geq G_\alpha = G_2$, then $TC(T_P^*) = TC_2(T_{P,2})$.
(b) If $G_\alpha = G_2 \geq 2K \geq G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_3(T_{P,3})\}$.
(c) If $2K \leq G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

(4) If $G_2 > G_\alpha > G_3$, then
(a) If $G_1 \geq 2K \geq G_2$, then $TC(T_P^*) = TC_2(T_{P,2})$.
(b) If $G_2 \geq 2K \geq G_\alpha$, then $TC(T_P^*) = TC_3(T_{P,3})$.
(c) If $G_\alpha \geq 2K \geq G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_3(T_{P,3})\}$.
(d) If $2K \leq G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

(5) If $G_\alpha = G_3$, then
(a) If $G_1 \geq 2K \geq G_2$, then $TC(T_P^*) = TC_2(T_{P,2})$.
(b) If $G_2 \geq 2K \geq G_\alpha = G_3$, then $TC(T_P^*) = TC_3(T_{P,3})$.
(c) If $2K \leq G_\alpha = G_3$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

(6) If $G_\alpha < G_3$, then
(a) If $G_1 \geq 2K \geq G_2$, then $TC(T_P^*) = TC_2(T_{P,2})$.
(b) If $G_2 \geq 2K \geq G_3$, then $TC(T_P^*) = TC_3(T_{P,3})$.
(c) If $G_3 \geq 2K \geq G_\alpha$, then $TC(T_P^*) = TC_6(T_{P,6})$.
(d) If $2K \leq G_\alpha$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

Case 2 When $M \geqslant \frac{W}{P-D} > \frac{D}{P}M$

When $M \geqslant \frac{W}{P-D} > \frac{D}{P}M$, total annual cost $TC(T_P)$ is

$$TC(T_P) = \begin{cases} TC_1(T_P), & \text{if } T_P \geqslant M, \\ TC_2(T_P), & \text{if } M \geqslant T_P \geqslant \dfrac{W}{P-D}, \\ TC_5(T_P), & \text{if } \dfrac{W}{P-D} \geqslant T_P \geqslant \dfrac{D}{P}M, \\ TC_6(T_P), & \text{if } T_P \leqslant \dfrac{D}{P}M. \end{cases}$$

Similar to the procedure in Case 1, we substitute Equation (28) into $T_P \geq M$ to obtain that $T_{P,1} \geqslant M$ if and only if

$$2K \geqslant \frac{M^2}{D}\left[(H + cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right] + M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{(P-D)D}. \tag{39}$$

Then, we have Lemma 1 discussed in Case 1.

We substitute Equation (31) into $M \geqslant T_P \geqslant \frac{W}{P-D}$ to obtain that $M \geqslant T_{P,2} \geqslant \frac{W}{P-D}$ if and only if

$$\begin{aligned} &\frac{M^2}{D}\left[(H + cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right] + M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{(P-D)D} \\ &\qquad \geqslant 2K \geqslant \frac{W^2}{D(P-D)^2}\left\{(h + cIp)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\} + DM^2(rIe - cIp). \end{aligned} \tag{40}$$

Let $G_4 = \frac{W^2}{D(P-D)^2}\left\{(h + cIp)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\} + DM^2(rIe - cIp)$, then we have the following lemma base on above analysis:

Lemma 5:

(a) If $2K > G_1$, there is no feasible solution in this case.
(b) If $G_1 \geq 2K \geq G_4$, there exists an optimal production run time $T_{P,2}$ (in Equation (31)).
(c) If $G_4 > 2K$, there is no feasible solution in this case.

The second-order derivative of $TC_5(T_P)$ with respect to T_P is

$$\frac{d^2TC_5(T_P)}{dT_P^2} = \frac{D[2K + DM^2(cIp - Ier)]}{PT_P^3} > 0. \tag{41}$$

There exists a production run time $T_{P,5}$ which minimises $TC_5(T_P)$ as follows. Solving $\frac{dTC_5(T_P)}{dT_P} = 0$, we obtain

$$T_{P,5} = \sqrt{\frac{D[2K - DM^2(Ier - cIp)]}{(h + cIp)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]}}. \tag{42}$$

Equation (42) gives the optimal value of $T_{P,5}$ for the case when $\frac{W}{P-D} \geqslant T_P \geqslant \frac{D}{P}M$. We substitute Equation (42) into $\frac{W}{P-D} \geqslant T_P \geqslant \frac{D}{P}M$ to obtain that $\frac{W}{P-D} \geqslant T_{P,5} \geqslant \frac{D}{P}M$ if and only if

$$\begin{aligned}&\frac{W^2}{D(P-D)^2}\left\{(h + cIp)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\} + DM^2(rIe - cIp)\\&\qquad\geqslant 2K \geqslant \frac{DM^2}{P^2}\left\{(h + rIe)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\}.\end{aligned} \tag{43}$$

Let $G_5 = \frac{DM^2}{P^2}\{(h + rIe)P^2 - D[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2]\}$, we have the following lemma base on above analysis:

Lemma 6:

(a) If $2K > G_4$, there is no feasible solution in this case.
(b) If $G_4 \geq 2K \geq G_5$, there exists an optimal production run time $T_{P,5}$ (in Equation (42)).
(c) If $G_5 > 2K$, there is no feasible solution in this case.

Equation (37) gives the optimal value of $T_{P,6}$ for the case when $T_P \leqslant \frac{D}{P}M$. We substitute Equation (37) into $T_P \leqslant \frac{D}{P}M$ to obtain that $T_{P,6} \leqslant \frac{D}{P}M$ if and only if

$$2K \leqslant \frac{DM^2}{P^2}\left\{(h + rIe)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\} \tag{44}$$

Then, we have the following lemma base on above analysis:

Lemma 7:

(a) If $2K > G_5$, there is no feasible solution in this case.
(b) If $2K \leq G_5$, there exists an optimal production run time $T_{P,6}$ (in Equation (37)).

From Equations (40) and (43), we know that $G_1 \geq G_4 \geq G_5$. Let $G_\alpha = M^2[cIp(P - D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, we have Theorem 2 based on above analysis:

Theorem 2:

(1) If $2K \geq G_1$, then $TC(T_P^*) = TC_1(T_{P,1})$.
(2) If $G_\alpha > G_4$, then
    (a) If $G_1 \geqslant 2K \geqslant G_4$, then $TC(T_P^*) = TC_2(T_{P,2})$.
    (b) If $G_4 \geq 2K \geq G_5$, then $TC(T_P^*) = Min\{TC_1(M), TC_5(T_{P,5})\}$.
    (c) If $2K \leq G_5$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.
(3) If $G_\alpha = G_4$, then
    (a) If $G_1 \geq 2K \geq G_\alpha = G_4$, then $TC(T_P^*) = TC_2(T_{P,2})$.
    (b) If $G_\alpha = G_4 \geq 2K \geq G_5$, then $TC(T_P^*) = Min\left\{TC_1(M), TC_5(T_{P,5})\right\}$.
    (c) If $2K \leq G_5$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.
(4) If $G_4 > G_\alpha > G_5$, then
    (a) If $G_1 \geq 2K \geq G_4$, then $TC(T_P^*) = TC_2(T_{P,2})$.
    (b) If $G_4 \geq 2K \geq G_\alpha$, then $TC(T_P^*) = TC_5(T_{P,5})$.
    (c) If $G_\alpha \geq 2K \geq G_5$, then $TC(T_P^*) = Min\{TC_1(M), TC_5(T_{P,5})\}$.
    (d) If $2K \leq G_5$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.
(5) If $G_\alpha = G_5$, then
    (a) If $G_1 \geq 2K \geq G_4$, then $TC(T_P^*) = TC_2(T_{P,2})$.

(b) If $G_4 \geq 2K \geq G_\alpha = G_5$, then $TC(T_P^*) = TC_5(T_{P,5})$.
(c) If $2K \leq G_\alpha = G_5$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

(6) If $G_\alpha < G_5$, then
(a) If $G_1 \geq 2K \geq G_4$, then $TC(T_P^*) = TC_2(T_{P,2})$.
(b) If $G_4 \geq 2K \geq G_5$, then $TC(T_P^*) = TC_5(T_{P,5})$.
(c) If $G_5 \geq 2K \geq G_\alpha$, then $TC(T_P^*) = TC_6(T_{P,6})$.
(d) If $2K \leq G_\alpha$, then $TC(T_P^*) = Min\{TC_1(M), TC_6(T_{P,6})\}$.

Case 3 When $\frac{W}{P-D} > M$

When $\frac{W}{P-D} > M$, total annual cost $TC(T_P)$ is

$$TC(T_P) = \begin{cases} TC_1(T_P), & \text{if } T_P \geqslant \dfrac{W}{P-D}, \\ TC_4(T_P), & \text{if } \dfrac{W}{P-D} \geqslant T_P \geqslant M, \\ TC_5(T_P), & \text{if } M \geqslant T_P \geqslant \dfrac{D}{P}M, \\ TC_6(T_P), & \text{if } T_P \leqslant \dfrac{D}{P}M. \end{cases}$$

Similar to the procedures in Case 1 and Case 2, we substitute Equation (28) into $T_P \geqslant \frac{W}{P-D}$ to obtain that $T_{P,1} \geqslant \frac{W}{P-D}$ if and only if

$$2K \geqslant \frac{W^2}{D(P-D)^2}\left\{(h+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\} + M^2[cIp(P-D) + DIer]. \tag{45}$$

Let $G_6 = \frac{W^2}{D(P-D)^2}\left\{(h+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\} + M^2[cIp(P-D) + Der]$. From $G_6 - \{M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}\} = \frac{W^2}{D(P-D)^2}\left[(H+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right]$ and λ is very small, we know that $G_6 > M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$. Then, we have the following lemma base on above analysis:

Lemma 8:

(a) If $2K \geq G_6$, there exists an optimal production run time $T_{P,1}$ (in Equation (28)).
(b) If $G_6 > 2K > M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, there is no feasible solution in this case.
(c) If $2K \leqslant M^2[cIp(P-D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$, the minimal value of $TC_1(T_P)$ occurs at $T_P = \frac{W}{P-D}$.

The second-order derivative of $TC_4(T_P)$ with respect to T_P is

$$\frac{d^2TC_4(T_P)}{dT_P^2} = \frac{D\left\{2K - M^2[cIp(P-D) + DIer]\right\}}{PT_P^3}. \tag{46}$$

If $2K > M^2[cIp(P-D) + DIer]$, $TC_4(T_P)$ is a convex function of T_P. Then, there exists a production run time $T_{P,4}$ which minimises $TC_4(T_P)$ as follows. Solving $\frac{dTC_4(T_P)}{dT_P} = 0$, we obtain

$$T_{P,4} = \sqrt{\frac{D\{2K - M^2[cIp(P-D) + DIer]\}}{(h+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2}}. \tag{47}$$

We substitute Equation (47) into $M \geqslant T_P \geqslant \frac{W}{P-D}$ to obtain that $M \geqslant T_{P,4} \geqslant \frac{W}{P-D}$ if and only if

$$\begin{aligned} &\frac{W^2}{D(P-D)^2}\left\{(h+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\} + M^2[cIp(P-D) + DIer] \\ &\qquad \geqslant 2K \geqslant \frac{M^2}{D}\left\{(h+cIp)P(P-D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\} + M^2[cIp(P-D) + DIer]. \end{aligned} \tag{48}$$

Let $G_7 = \frac{M^2}{D}\left\{(h + cIp)P(P - D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\} + M^2[cIp(P - D) + DIer]$. From $G_7 - M^2[cIp(P - D) + DIer] = \frac{M^2}{D}\left\{(h + cIp)P(P - D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\}$ and λ is very small, we know that $G_7 > M^2[cIp(P - D) + DIer]$. Then,

Lemma 9:

(a) If $2K > G_6$, there is no feasible solution in this case.
(b) If $G_6 \geq 2K \geq G_7$, there exists an optimal production run time $T_{P,4}$ (in Equation (47)).
(c) If $G_7 \geq 2K \geq M^2[cIp(P - D) + DIer]$, there is no feasible solution in this case.
(d) If $2K \leq M^2[cIp(P - D) + DIer]$, the minimal value of $TC_4(T_P)$ occurs at $T_P = M$.

Equation (42) gives the optimal value of $T_{P,5}$ for the case when $M \geqslant T_P \geqslant \frac{D}{P}M$. We substitute Equation (42) into $M \geqslant T_P \geqslant \frac{D}{P}M$ to obtain that $M \geqslant T_{P,5} \geqslant \frac{D}{P}M$ if and only if

$$\frac{M^2}{D}\left\{(h + cI_p)P(P - D) + DPs(\alpha_O - \alpha_I)\lambda - Dm\lambda^2\right\} + M^2[cI_p(P - D) + DI_e r] \geqslant 2K \geqslant \frac{DM^2}{P^2}\left\{(h + rI_e)P^2 - D[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2]\right\} \tag{49}$$

Then, we have the following lemma base on above analysis:

Lemma 10:

(a) If $2K > G_7$, there is no feasible solution in this case.
(b) If $G_7 \geq 2K \geq G_5$, there exists an optimal production run time $T_{P,5}$ (in Equation (42)).
(c) If $G_5 > 2K$, there is no feasible solution in this case.

Equation (37) gives the optimal value of $T_{P,6}$ for the case when $T_P \leqslant \frac{D}{P}M$. We substitute Equation (37) into $T_P \leqslant \frac{D}{P}M$ to obtain that $T_{P,6} \leqslant \frac{D}{P}M$ if and only if

$$2K \leqslant \frac{DM^2}{P^2}\left\{(h + rI_e)P^2 - D\left[hP - Ps(\alpha_O - \alpha_I)\lambda + m\lambda^2\right]\right\}. \tag{50}$$

Then, we have Lemma 7 discussed in Case 2.

From Equation (48) and (49), we know that $G_6 \geq G_7 \geq G_5$. Let $G_\alpha = M^2[cIp(P - D) + DIer] - \frac{(H-h)PW^2}{D(P-D)}$ and $G_\beta = M^2[cIp(P - D) + DIer]$. From $G_\beta - G_\alpha = \frac{(H-h)PW^2}{D(P-D)}$ we know that $G_\beta > G_\alpha$. Then we have Theorem 3 based on above analysis:

Theorem 3:

(1) If $2K \geq G_6$, then $TC(T_P^*) = TC_1(T_{P,1})$.
(2) If $G_7 > G_\beta > G_\alpha > G_5$, then
  (a) If $G_6 \geq 2K \geq G_7$, then $TC(T_P^*) = TC_4(T_{P,4})$.
  (b) If $G_7 \geq 2K \geq G_\beta$, then $TC(T_P^*) = TC_5(T_{P,5})$.
  (c) If $G_\beta \geq 2K \geq G_\alpha$, then $TC(T_p^*) = TC_5(T_{P,5})$.
  (d) If $G_\alpha \geq 2K \geq G_5$, then $TC(T_P^*) = Min\left\{TC_1(\frac{W}{P-D}), TC_5(T_{P,5})\right\}$.
  (e) If $2K \leq G_5$, then $TC(T_P^*) = Min\left\{TC_1(\frac{W}{P-D}), TC_4(M), TC_6(T_{P,6})\right\}$.
(3) If $G_7 > G_\beta > G_\alpha = G_5$, then
  (a) If $G_6 \geq 2K \geq G_7$, then $TC(T_P^*) = TC_4(T_{P,4})$.
  (b) If $G_7 \geq 2K \geq G_\beta$, then $TC(T_P^*) = TC_5(T_{P,5})$.
  (c) If $G_\beta \geq 2K \geq G_5 = G_\alpha$, then $TC(T_P^*) = Min\left\{TC_4(M), TC_5(T_{P,5})\right\}$.
  (d) If $2K \leq G_5 = G_\alpha$, then $TC(T_P^*) = Min\left\{TC_1(\frac{W}{P-D}), TC_4(M), TC_6(T_{P,6})\right\}$.
(4) If $G_7 > G_5 = G_\beta > G_\alpha$, then
  (a) If $G_6 \geq 2K \geq G_7$, then $TC(T_P^*) = TC_4(T_{P,4})$.
  (b) If $G_7 \geq 2K \geq G_5 = G_\beta$, then $TC(T_P^*) = TC_5(T_{P,5})$.
  (c) If $G_5 = G_\beta \geq 2K \geq G_\alpha$, then $TC(T_P^*) = Min\left\{TC_4(M), TC_6(T_{P,6})\right\}$.
  (d) If $2K \leq G_\alpha$, then $TC(T_P^*) = Min\left\{TC_1(\frac{W}{P-D}), TC_4(M), TC_6(T_{P,6})\right\}$.

(5) If $G_7 > G_5 > G_\beta > G_\alpha$, then
   (a) If $G_6 \geq 2K \geq G_7$, then $TC(T_P^*) = TC_4(T_{P,4})$.
   (b) If $G_7 \geq 2K \geq G_5$, then $TC(T_P^*) = TC_5(T_{P,5})$.
   (c) If $G_5 \geq 2K \geq G_\beta$, then $TC(T_P^*) = TC_6(T_{P,6})$.
   (d) If $G_\beta \geq 2K \geq G_\alpha$, then $TC(T_P^*) = Min\{TC_4(M), TC_6(T_{P,6})\}$.
   (e) If $2K \leq G_\alpha$, then $TC(T_P^*) = Min\{TC_1(\frac{W}{P-D}), TC_4(M), TC_6(T_{P,6})\}$.

(6) If $G_7 > G_\beta > G_5 > G_\alpha$, then
   (a) If $G_6 \geq 2K \geq G_7$, then $TC(T_P^*) = TC_4(T_{P,4})$.
   (b) If $G_7 \geq 2K \geq G_\beta$, then $TC(T_P^*) = TC_5(T_{P,5})$.
   (c) If $G_\beta \geq 2K \geq G_5$, then $TC(T_P^*) = Min\{TC_4(M), TC_5(T_{P,5})\}$.
   (d) If $G_5 \geq 2K \geq G_\alpha$, then $TC(T_P^*) = Min\{TC_4(M), TC_6(T_{P,6})\}$.
   (e) If $2K \leq G_\alpha$, then $TC(T_P^*) = Min\left\{TC_1(\frac{W}{P-D}), TC_4(M), TC_6(T_{P,6})\right\}$.

Therefore, the optimal production run time can be determined by the following algorithm.

Algorithm:

Step 1 When $M > \frac{D}{P}M \geqslant \frac{W}{P-D}$, go to Step 2; when $M \geqslant \frac{W}{P-D} > \frac{D}{P}M$, go to Step 3; otherwise go to Step 4.

Step 2 Use Theorem 1 to determine the optimal T_P^* and $TC(T_P^*)$.

Step 3 Use Theorem 2 to determine the optimal T_P^* and $TC(T_P^*)$.

Step 4 Use Theorem 3 to determine the optimal T_P^* and $TC(T_P^*)$.

One purpose of this paper is to extend Tsao (2009) to the case that the warehouse space is limited. Here, we demonstrate that the model in Tsao (2009) is a special case in our model. When $W \to \infty$ and $H = h$, Case 1 and Case 2 in our model do not exist. And Case 3 becomes

Table 1. Effects of K when $M > \dfrac{D}{P}M \geqslant \dfrac{W}{P-D}$.

	Values in					
Changing parameter	G_α	G_1	G_2	G_3	T_P^*	$C(T_P^*)$
$K = 300$	208.75	566.05	146.2	62.2	0.3139	748.15
$K = 200$	208.75	566.05	146.2	62.2	0.2516	569.98
$K = 50$	208.75	566.05	146.2	62.2	0.1250	170.00
$K = 30$	208.75	566.05	146.2	62.2	0.0980	80.45

Table 2. Effects of K when $M \geqslant \dfrac{W}{P-D} > \dfrac{D}{P}M$.

	Values in					
Changing parameter	G_α	G_1	G_4	G_5	T_P^*	$TC(T_P^*)$
$K = 300$	193.75	551.05	240.05	139.95	0.3199	734.98
$K = 200$	193.75	551.05	240.05	139.95	0.2564	559.75
$K = 80$	193.75	551.05	240.05	139.95	0.1613	273.71
$K = 30$	193.75	551.05	240.05	139.95	0.0980	80.45

Table 3. Effects of K when $\dfrac{W}{P-D} > M$.

	Values in					
Changing parameter	G_α	G_6	G_7	G_5	T_P^*	$TC(T_P^*)$
$K = 400$	133.75	768.95	526.05	139.95	0.4096	863.175
$K = 300$	133.75	768.95	526.05	139.95	0.3336	728.854
$K = 80$	133.75	768.95	526.05	139.95	0.1613	273.708
$K = 30$	133.75	768.95	526.05	139.95	0.0980	80.45

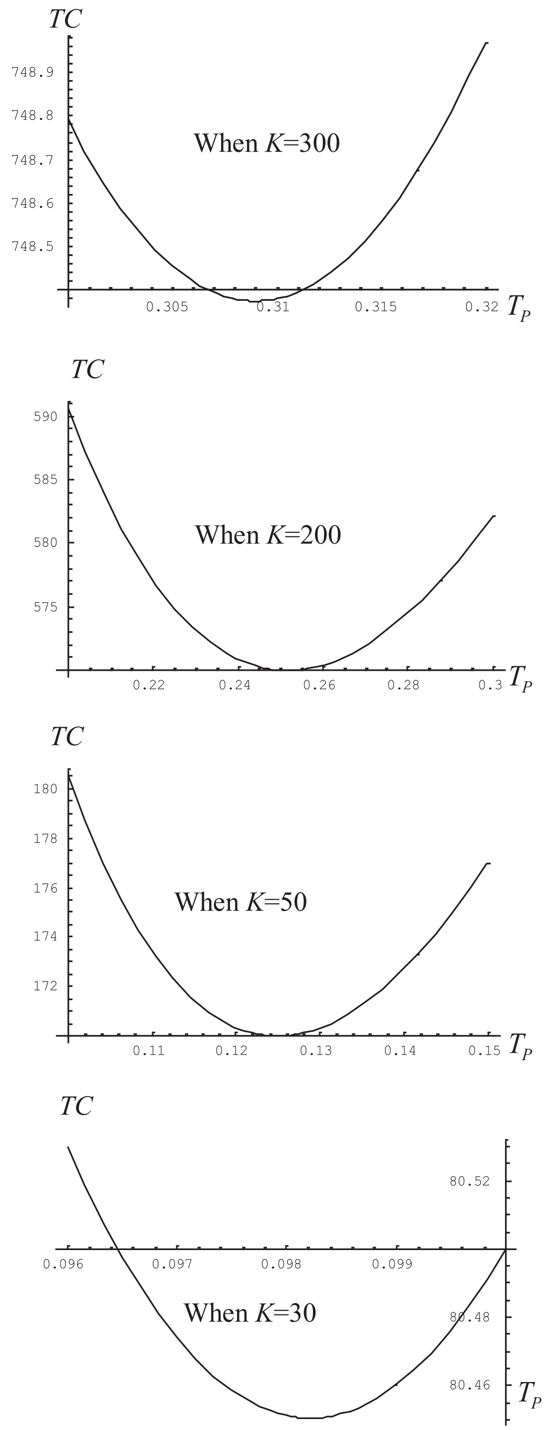

Figure 2. Graphic illustration of TC vs. T_P when $M > \frac{D}{P}M \geqslant \frac{W}{P-D}$.

$$TC(T_P) = \begin{cases} TC_7(T_P), & \text{if } T_P \geqslant M, \\ TC_8(T_P), & \text{if } M \geqslant T_P \geqslant \dfrac{D}{P}M, \\ TC_9(T_P), & \text{if } T_P \geqslant \dfrac{D}{P}M, \end{cases} \tag{51}$$

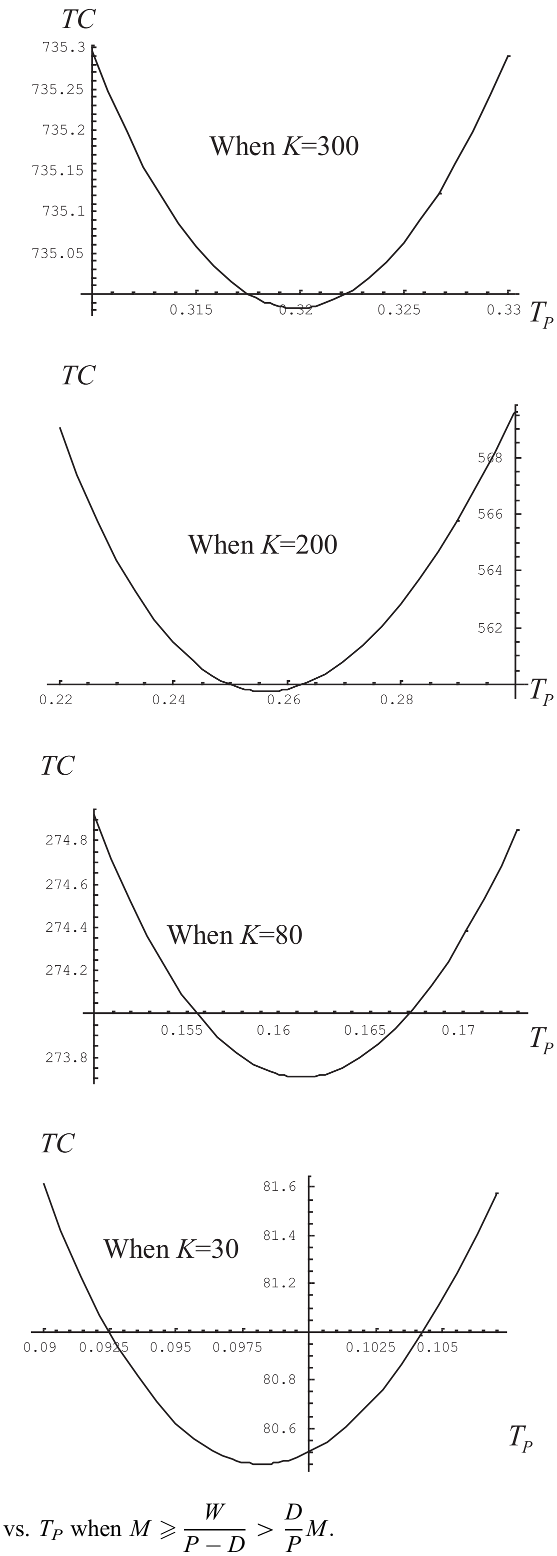

Figure 3. Graphic illustration of TC vs. T_P when $M \geqslant \dfrac{W}{P-D} > \dfrac{D}{P}M$.

where

$$TC_7(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P}$$
$$+ c \cdot Ip \cdot \left[\frac{PT_P}{2}\left(1 - \frac{D}{P}\right) - \frac{(P-D)D \cdot M^2}{2PT_P}\right], \quad (52)$$

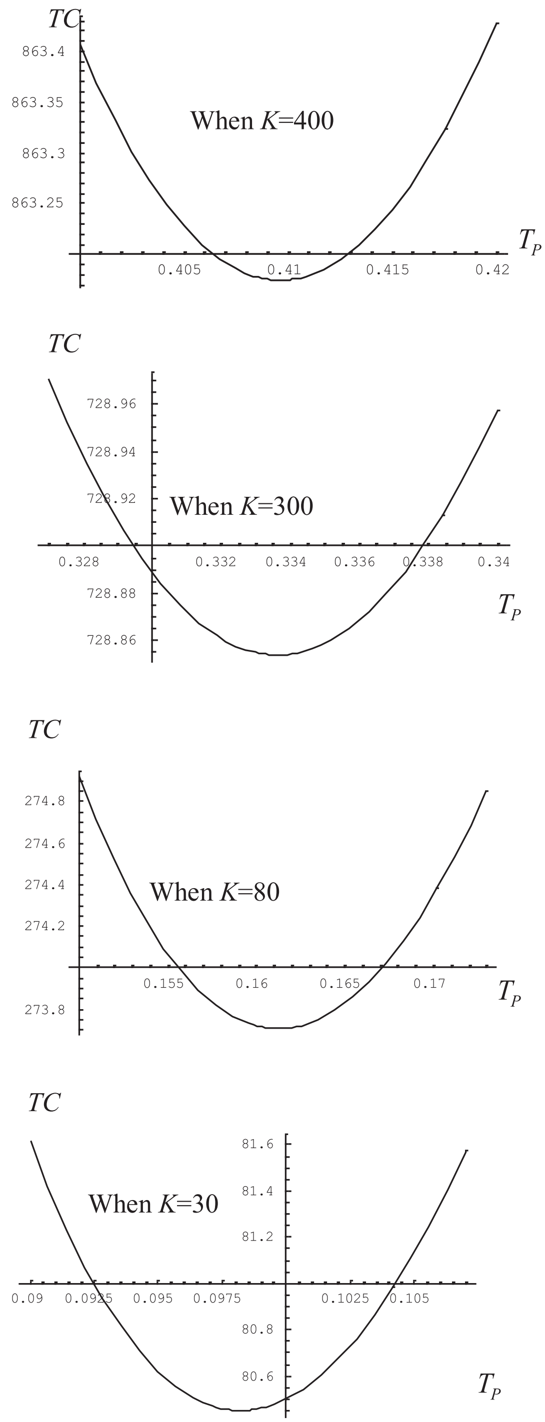

Figure 4. Graphic illustration of TC vs. T_P when $\dfrac{W}{P-D} > M$.

$$TC_8(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot D^2 \cdot M^2}{2PT_P} + \frac{c \cdot Ip \cdot D^2 \cdot (PT_P/D - M)^2}{2PT_P}, \tag{53}$$

$$TC_9(T_P) = \frac{KD}{PT_P} + \frac{h(P-D)T_P}{2} + \frac{mD}{P}\left(\lambda - \frac{\lambda^2 T_P}{2}\right) + sD\left[\alpha_I + (\alpha_O - \alpha_I)\frac{\lambda T_P}{2}\right] - \frac{r \cdot Ie \cdot P \cdot T_P}{2} - r \cdot Ie \cdot D \cdot \left(M - \frac{P \cdot T_P}{D}\right). \tag{54}$$

Equations (52)–(54) are consistent with Equations (4)–(6) in Tsao (2009), respectively. Hence, Tsao (2009) is a special case of this paper when the warehouse space limitation is not considered.

5. Computational analysis

We use the following example to illustrate the proposed model: $r = 25$, $c = 1.5$, $D = 500$, $p = 1000$, $h = 1$, $H = 1.5$, $s = 5$, $m = 500$, $\lambda = 0.1$, $\alpha_I = 0.05$, $\alpha_O = 0.5$, $Ie = 0.1$, $Ip = 0.15$, $M = 0.3$. Table 1 is for $W = 50$ (Case 1: When $M > \frac{D}{P}M \geqslant \frac{W}{P-D}$), Table 2 is for $W = 100$ (Case 2: When $M \geqslant \frac{W}{P-D} > \frac{D}{P}M$), and Table 3 is for $W = 200$

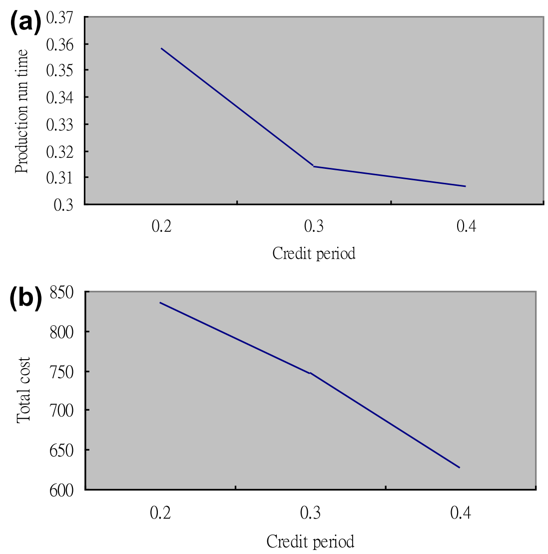

Figure 5. Effects of credit period.

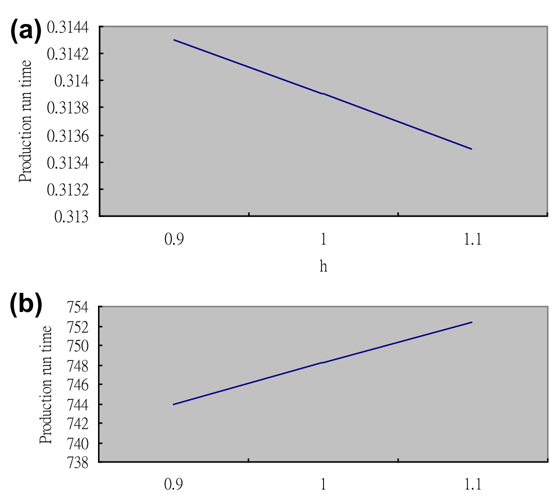

Figure 6. Effects of inventory holding cost in manufacturer's warehouse.

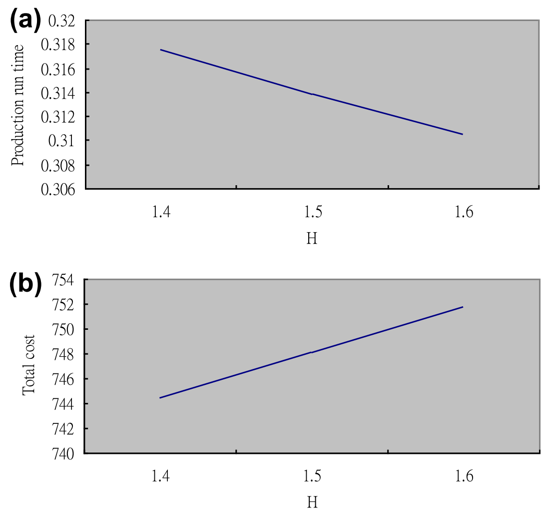

Figure 7. Effects of inventory holding cost in rented warehouse.

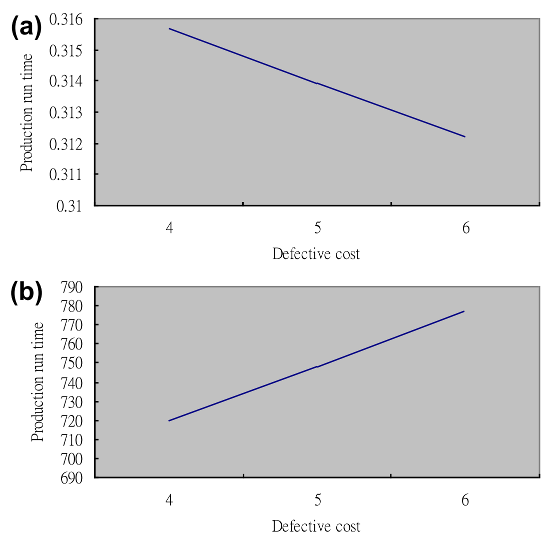

Figure 8. Effects of defective cost.

(Case 3: When $\frac{W}{P-D} > M$). Applying the Algorithm, the results are shown in Tables 1–3. Graphic representations of TC for each case are shown in Figures 2–4. We summarise the result shown in Tables 1–3 as follows:

(1) When the manufacturer need to rent the warehouse to store the exceeding items, the optimal production run time T_P^* will increase and the total cost $TC(T_P^*)$ will decrease as the manufacturer's warehouse capacity W increases. This means the manufacturer want to store more items in his own warehouse to decrease the cost if he has larger warehouse capacity.

(2) The optimal production run time T_P^* and the total cost $TC(T_P^*)$ will increase as the set-up cost K increases. This means the manufacturer want to increase the production run time to decrease the set-up times Then, the total cost can be decreased.

It is also important to investigate the effects of credit period M, inventory holding cost in manufacturer's warehouse h, inventory holding cost in rented warehouse H, defective cost s and machine maintenance cost m on decisions and costs. Following most values of the parameter setting in Case 1, Figures 5–9 present the following numerical results:

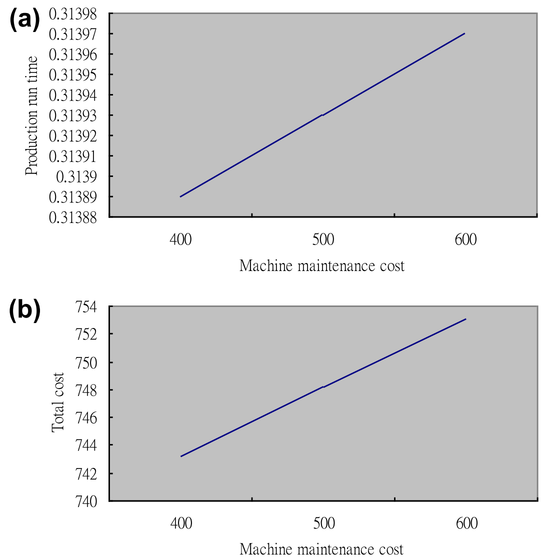

Figure 9. Effects of machine maintenance cost.

(1) When the credit period M increases or the machine maintenance cost m decreases, both the optimal production run time T_P^* and the total cost $TC(T_P^*)$ will decrease. When the credit period increases, the manufacturer will decrease the cycle time to get more benefit from trade credit.
(2) When the inventory holding cost in manufacturer's warehouse h, inventory holding cost in rented warehouse H, or defective cost s increases, the optimal production run time T_P^* will decrease and the total cost $TC(T_P^*)$ will increase. When the inventory holding cost increases, the manufacturer should decrease the production run time to lower inventory costs.

6. Conclusions

This paper considers the optimal production policy for an imperfect manufacturing system with machine maintenance activities, trade credit and limited warehouse space. Machine maintenance activities, trade credit and limited warehouse space are commonly in small and medium size manufacturers. This paper relaxes several assumptions of EPQ model to cope with more general and practical situations. The objective is to determine the optimal production run time to minimise the total cost. This paper provides an algorithm based on several theorems for solving the problem. From computational analysis, we discuss how the system parameters affect the manufacturer's decision and cost. The purpose of this paper is to relax several assumptions to cope with more practical situations.

The contributions of this paper to the literature are as follows. First, this paper is the first to incorporate the limited warehouse space, trade credit and system maintenance into EPQ model simultaneously. The complex problem is modelled as a piecewise nonlinear model which is not easy to handle We show that the model in previous research is a special case in our model. Second, this paper develops several theorems based on lemmas for optimisation. And provide an algorithm based on these theorems for solving the problem. Third, we perform numerical analysis to study the effects of changing parameter values on the optimal solutions and to point out some management implications. The results of this study are a useful reference for managerial decision-making and administration.

This paper deals with the production decision from the perspective of cost, and can be used as an add-in optimiser in an Enterprise Resource Planning (ERP) system. For example, Advanced Planning and Optimiser (APO) is the optimiser in SAP. In APO, several optimisation techniques such as Linear Programme, Integer Programme, Nonlinear Programme, Genetic Algorithms, Tabu Search, etc. are used to solve realistic problems. Our model utilises the nonlinear programme to solve the production-inventory problem. Thereby, one can embed the model and solution procedure developed in this paper in an ERP system to determine the production-inventory decision. For the further research, this paper can be extended to consider other situations, such as for stochastic demand, deteriorating items or shortage allowance, etc.

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